

## FINITE ELEMENT BUCKLING ANALYSIS OF CRACKED CYLINDRICAL SHELLS UNDER TORSION

H. E. Estekanchi, A. Vafai and K. Kheradmandnia  
Department of Civil Engineering, Sharif University of Technology, Tehran, Iran

### ABSTRACT

Buckling strength and imperfection sensitivity are major considerations in the analysis and design of structural shell forms. In this paper, the buckling load and its sensitivity to through cracks with variable length and orientation, and buckling mode shapes of cylindrical shells under torsional loading are studied. A general finite element model has been proposed and capability of this model in predicting the torsional buckling load has been verified. The model is then applied to the analysis of cylindrical shells with circumferential, axial and angled cracks of various lengths. The results are presented in the form of tables and normalized figures.

**Keywords:** cracked shells, shell buckling under torsion, finite element buckling analysis

### 1. INTRODUCTION

Plate and shell structures have a wide use in the new technological fields of civil and mechanical engineering, such as aerospace and marine structures, large dams, tall silos and long span roofs. The development of construction technology and new materials such as composites, which are light, firm and economical, has stimulated vast research on shell structures [1-3]. One of the greatest concerns of structural engineers, is the safety reliability of such structures during utilization period. Shell structures, like many other types of structures, are subjected to damages ranging from corrosion, chemical rushes and erosion, to initiation and expansion of cracks. The effects of these damages are very critical because in shell structures buckling behavior determines load carrying capacity and this behavior is very sensitive to imperfections [4-6]. There is also the question of how a crack, as an initial imperfection, affects the load carrying capacity of a shell structure. Though it is well known that the presence of cracks or similar imperfections can considerably reduce the buckling load of a shell structure, there are no definite guidelines for structural engineers to estimate the effects of such imperfections and the decrease of load carrying capacity of a cracked shell structure.

In this research, the buckling sensitivities of cylindrical shells under torsional loading, when subjected through cracks with various parameters such as crack length, crack angle, edge conditions and Poisson's ratio are studied. A general finite element model has been proposed and its behavior in estimating the torsional buckling load has been determined in the case of non-cracked shells. After the assessment of model correctness, the behavior of some cracked shells, for which documented results are not available, is studied. These studies include some more than

pressure and tension [15].

### 3. THEORETICAL BUCKLING LOAD ANALYSIS

The objective of Solid Mechanics is to define the deformation equations of the body in such a way to satisfy the equilibrium equations. As it is known that the stress depends on strain, and the strain depends on deformation, a series of second order differential equations must be solved.

These equations can only be solved with accuracy for simple geometric conditions and loading, but in the cases of complicated geometric and general loading and boundary conditions, these equations are not applicable and instead numerical and approximate methods should be used. Such approximate solutions, like Finite Elements Method analysis, incorporate energy method, or in the other words, the principle of minimum potential energy. This principle states that for a conservative system, among all deformation cases which satisfy boundary condition, that deformation meets the equilibrium conditions that maximizes/minimizes the potential energy of the total system and if this desired deformation minimizes the potential energy, the equilibrium of the body will remain stable. The total potential energy of an elastic body is shown by  $\Pi$ , we have:

$$\Pi = U + V \quad (1)$$

In this equation,  $U$  is the strain energy of the body for an elastic material which results from the following formula:

$$U = \frac{1}{2} \int_V \sigma^T \epsilon dV \quad (2)$$

and  $V$ , means the potential work or accomplishment of external loading. With the assumption of conservation of applied load:

$$V = - \int_V u^T f dV - \int_S u^T T dS - \sum_{i=1}^n u_i^T P \quad (3)$$

In the above equation,  $n$  is the number of point loads exerted on the body.

The purpose of calculating the buckling load is to estimate the maximum load a structure can carry before instability. The most exact procedure to calculate the buckling load of a structure, is the nonlinear analysis with incremental loading, while the structure itself buckles at the point where the slope of load-deformation curve becomes zero. However, this procedure is very time consuming and expensive, needing advanced computer programs. Furthermore, for practical purposes, an eigen value analysis will work with good precision. The equation for computing the eigen value of the stiffness matrix is:

$$(K + \lambda_i K_g) u_i = 0 \quad (4)$$

In the above equation,  $\lambda_i$  is the eigen amount of  $i$ , or in other words, the buckling load factor of  $i$ ;  $u_i$  is the eigen vector of  $i$  or deformation in mode  $i$ , and  $K_g$  is the geometric or initial stiffness matrix.

#### 4. THE STUDY OF EXISTING THEORIES

Here the theories and existing theoretical relations generally presented for calculating non-cracked cylindrical shells buckling load are studied. The existing theoretical formulas are used for comparison with the numerical results of the computer. For instance, the most famous existing theory in this field is Danel's nonlinear theory. This theory was made by Danel in 1933, in relation to torsional buckling analyze of the short shells with thin walls. Because of its relative simplicity and fine practical accuracy, this theory has gained wide use in solving the buckling problems and after-buckling phenomenon.

Danel's Formula for calculating the cylindrical shells buckling load with  $Z$  more than 10,000 is as the following:

$$\frac{\tau_c}{E} = \frac{0.925\pi}{12(1-\nu^2)} (1-\nu^2)^{3/8} \left(\frac{t}{r}\right)^{5/4} \left(\frac{r}{L}\right)^{1/4} \quad \& \quad Z = \sqrt{1-\nu^2} \frac{L^2}{rt} \quad (5)$$

$$n = 1.35\pi (1-\nu^2)^{1/8} \left(\frac{r}{L}\right)^{1/2} \left(\frac{r}{t}\right)^{1/4} \quad (6)$$

For  $z$  less than 10,000, buckling load factor is calculated numerically and mentioned in the related tables. The other important theories are Flugge and Sanders-Quiter's [16,17]. Timoshenko in his book used the general theory of shells made by A.E. Love to present the following formula for the calculation of the buckling load of a cylindrical shell under torsion [18]:

$$(1-\nu^2) \frac{\tau_{cr}}{E} \frac{L^2}{t^2} = 4.6 + \sqrt{7.8 + 1.67 \left( \sqrt{1-\nu^2} \frac{L^2}{2rt} \right)^{3/2}} \quad (7)$$

As an alternative, the following formula has been proposed by other researchers [19]:

$$(1-\nu^2) \frac{\tau_{cr}}{E} \frac{L^2}{t^2} = -2.39 + \sqrt{96.9 + 0.605 \left( \sqrt{1-\nu^2} \frac{L^2}{rt} \right)^{3/2}} \quad (8)$$

It should be mentioned that to calculate the critical torsional Moment from the above formulas, the following equation can be used:

$$M_{cr} = 2\pi r^2 t \tau_{cr} \quad (9)$$



## 5. MAKING THE COMPUTERIZED MODEL AND THE ASSESSMENT OF ITS ACCURACY

To make the computerized model and assess its accuracy, the effective parameters in numerical results obtained from computer for static and torsional loading will be studied. In order to check the numerical results obtained from computer, non-cracked models will be used. In this case, there would be intermediary theories to compare the numerical and theoretical results. Among the studied cases in this section are the effect of element size, shell thickness, shell length and Poisson's ratio for calculating the buckling load and the static study of cylindrical shells under torsional load.

### 5.1 Suitable element size for calculating the buckling load

Considering the shape of the shell and the complexity of the case, it is concluded that to get the proper result, a very small element mesh is required. This was proved after some experimental model analyzing and observation of numerical accuracy. The element sizes must be such to cover the buckling shape of the shell.

If the accuracy of the element is less than a certain amount, some buckling modes that have many waves (especially circumferential waves) will be eliminated and computer program will converge to upper modes. This subject especially in regard to cylindrical shells which have much close buckling modes with short circumferentially waves, is highly important. To make sure of the appropriate number of considered elements, shells with different amounts of elements will be studied. First, we will start with a small amount of circumferential elements and gradually will decrease the element mesh to study the process of results convergence to the theoretical amount. It is obvious that in the case of shell element, the greatest accuracy will be obtained when the element is square or close to square. Besides, there is no need to use rectangular-shaped elements. In all cases, it has been tried to arrange the number of circumferential elements in shell and shell height, such that the element size would be as close to square as possible. The structure used as a basis was an aluminum cylindrical shell with medium height. The critical torsional moment of this shell ( $M_{cr}$ ) for non-cracked condition, can be obtained from different theoretic formulas as the follows:

	t = 1mm	t = 2mm
Donnel's theory	18,661	88,349
Reference [18]	19,091	91,267
Reference [19]	19,115	90,711

General shell geometry and plan view of buckled shape of models with 22 and 36 circumferential elements are drawn in Figures 1 and 2 respectively. These shapes are important because they help to distinguish the number of circumferential waves and to illustrate the symmetry of buckled shapes. As the study of buckling shapes shows, the number of circumferential elements, even less than 30, makes no problem for the formation of the buckled shape and the number of buckled-shape circumferential waves corresponds to that mentioned in Danel's theory. A study of convergence of buckling load for analyzed models shows that with an increase in the number of elements, the numerical results will gradually converge to the theoretical amount. A reasonable number of circumferential elements should be considered in order to have both acceptable accuracy and economy of time based on convergence tests. With

respect to the buckling mode shapes, buckling load scale chart and the efficient time for calculating each model, the number of the elements on the cylindrical circumference ( $2^2 \times 3^2 = 36$ ) will be chosen. Therefore, on each boundary edge, we have 72 nodes. This number of elements gives acceptable accuracy (less than 5% fault in worst cases) in calculating the buckling load of the shell.

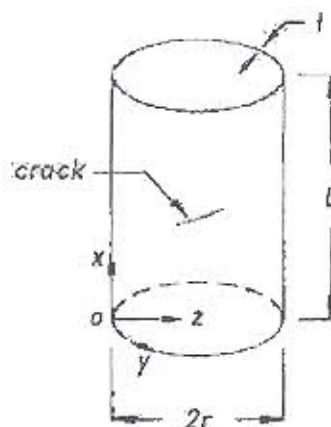


Figure 1 Basic cylinder dimensions and coordinate system.

### 5.2. Comparison with the static behavior of non-cracked model

In the next step, in order to make sure of the model accuracy, the behavior of the non-cracked cylindrical static model would be studied. For this purpose, an uncracked cylindrical model with 2mm thickness under torsional moment equal to 1000 Newton-meter is considered.

The model's upper cap rotation angle when subjected to load is  $4.729 \times 10^{-5}$  radian which is exactly equal to the theoretical amount. In this model, the amount of shearing stress per unit thickness is equal to 636.620 Newton/Meter which is also exactly the amount of  $N_{xy}$  obtained from the theory. It can be concluded that the model is adequate for modeling the stress and strain fields in static torsional loading

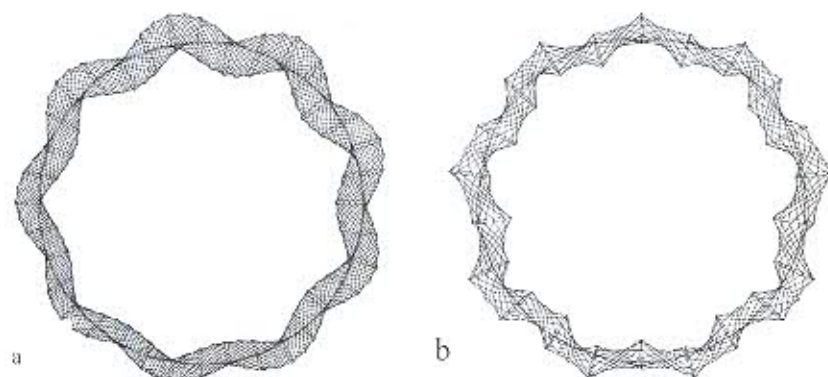


Figure 2 Buckled shape of cylinders with 36 and 22 circumferential elements

## 6. THE STUDY OF CRACKED CYLINDRICAL SHELL BEHAVIOR

### 6.1 Static behavior of cracked cylindrical shell

In this section, the static behavior of models with horizontal, vertical and angled cracks is studied. In all of these models, crack length is 0.7m. Typical pre-buckling elastic deformation shape of models with axial and circumferential crack under torsional moment is depicted in Figure 3. The model's upper cap rotation angle for cylindrical shells with different cracks are as follows:

Shell with circumferential crack	$6.996 \times 10^{-5}$
Shell with Axial-crack	$5.182 \times 10^{-5}$
Shell with Angled-crack	$6.517 \times 10^{-5}$
Shell with no crack	$4.729 \times 10^{-5}$

Considering the above results, it seems that the shell with axial-crack undergoes less deformation than other cases, thus having less stiffness reduction.

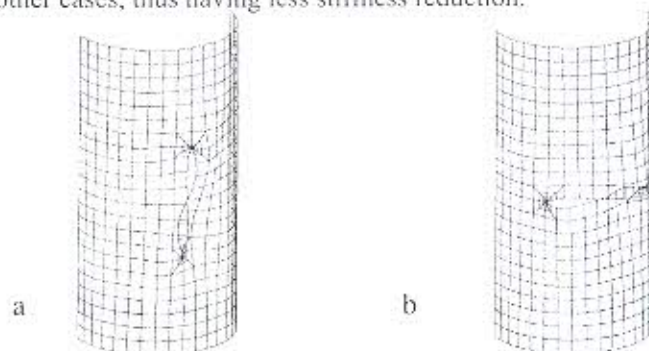


Figure 3 Elastic deformation of shells with axial and circumferential cracks

### 6.2 Buckling behavior of circumferentially cracked shells

The behavior of circumferentially cracked cylindrical shells under torsion is studied. The general specifications of these shells are the same as non-cracked shells studied in the previous section. A circumferential crack with desired length is produced on a shell and is increased gradually. In each stage, the normalized value of reduction of cracked shell buckling capacity is calculated.

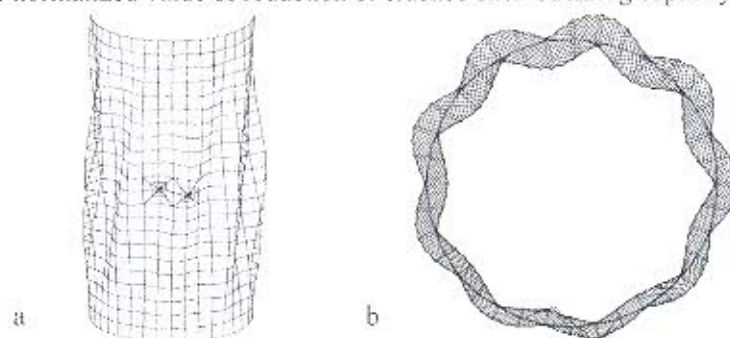


Figure 4 Front and plan view of the buckled shape of circumferentially cracked shell (crack length is less than critical crack length).



Buckled shape of circumferentially cracked shell with short and long crack is shown in Figures 4 and 5. As it can be seen, the increase of crack length up to a certain limit, doesn't affect the buckled shape of the cylinder, in other words, the buckling mode of the cracked shell remains the same as that of the non-cracked shell.

In order to study the effect of thickness on buckling load, each model is analyzed with four different thicknesses. The normalized ratio of cracked model buckling load to non-cracked model buckling load (Reduction Factor or RF), with respect to crack length, is presented in Figure 6. The increase in the crack length, to a certain limit which is a property of shell specification, has no effect on buckling load. In fact, when the crack length is less than a certain limit, the buckling load of cracked shell is equal to that of the non-cracked shell. In addition, it can be seen that the thickness ratio ( $r/t$ ) has little effect on the reduction of buckling load.

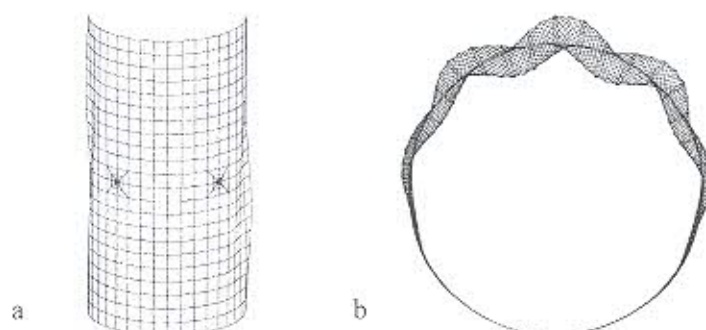


Figure 5 Front and plan view of the buckled shape of circumferentially cracked shell (crack length is greater than critical crack length).

In order to use the results of this section more conveniently, it is tried to fit a suitable formula into the above curves. After testing various formulas, the following equation is suggested to obtain the ratio of the buckling load of cracked shell to non-cracked shell under torsional loading:

$$RF = 1.623 \left[ 1 - 0.338 \left( \frac{a}{r} \right)^{0.48} \right] \left( \frac{r}{t} \right)^{-0.049} \leq 1 \quad (10)$$

$$0 \leq \frac{a}{r} \leq 3.5 \quad \& \quad 125 \leq \frac{r}{t} \leq 1000$$

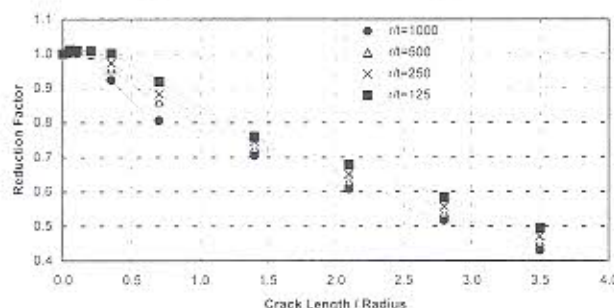


Figure 6 Reduction factor for shells with circumferential crack.

### 6.3 Buckling behavior of axially cracked shells

The behavior of axially cracked cylindrical shells under torsion is studied in this section. The general specifications of these shells are the same as non-cracked shells studied in the previous section. An axial crack with desired length is produced on a shell and is increased gradually. In each stage, the normalized value of the reduction of cracked shell buckling capacity is calculated.

To study the effect of thickness on buckling load, each model is analyzed with four different thicknesses. The normalized ratio of cracked model buckling load to non-cracked model buckling load, according to the ratio of crack length to cylinder radius, is presented in Figure 7. The increase in the crack length, to a certain limit which is a property of shell specification has no effect on buckling load. In fact, when the crack length is less than a certain limit, the buckling load of cracked shell is equal to that of the non-cracked shell. As can be seen, the effect of thickness ratio is more pronounced in this case. This figure also shows that for a cylindrical shell with a known crack length, with reduction in thickness, the buckling load decreases in comparison to the buckling load of the non-cracked shell. Buckling shape plan of axially cracked shell in three cases (crack length greater/less than and near to the critical crack length is shown in Figures 8 and 9. As it can be seen, the increase of crack length up to a certain limit, doesn't affect the buckled shape of the cylinder, in other words, the buckling mode shape of the cracked shell remains the same as that of the non-cracked shell. When the crack length exceeds the critical amount, the buckling mode shape of the shell won't be the global buckling mode and the shell will show local buckling in crack edges. This matter, will cause a high decrease in the buckling load to 1/2 of the non-cracked cylindrical shell buckling load. Thereafter, the buckling load will have little sensitivity to crack length and with increase in the crack length, we will have a small decrease in the buckling load.

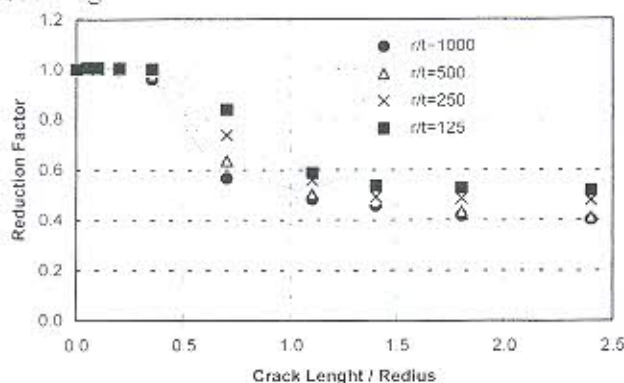


Figure 7 Reduction factor for shells with axial crack

To use the results of this section more conveniently, it is tried to fit a suitable formula into the above curves. After testing various formulas, the following equation is suggested to obtain the ratio of the buckling load of cracked shell to non-cracked shell under torsional loading:

$$RF = 0.842 \left[ 0.219 + 0.655 \left( \frac{a}{r} \right) \right] \left( \frac{r}{t} \right)^{-0.123} \leq 1 \quad (11)$$



$$0 \leq \frac{a}{r} \leq 2.4 \quad \& \quad 125 \leq \frac{r}{t} \leq 1000$$

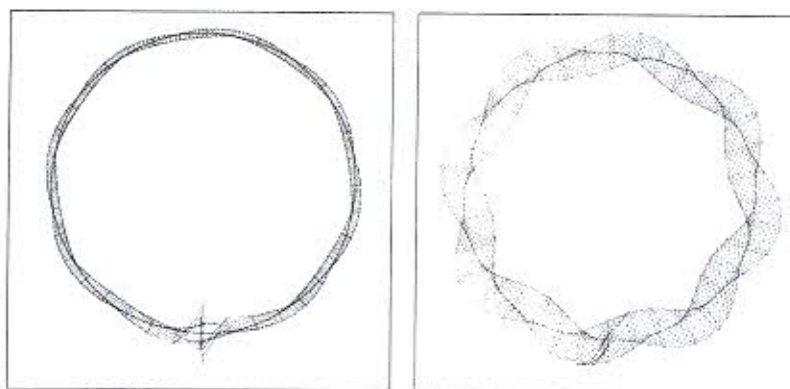


Figure 8 Plan view of the buckled shape of axially cracked shell with near critical and short cracks



Figure 9 Plan view of the buckled shape of axially cracked shell with long crack

#### 6.4 Shells with angled cracks and other cases studied in this research

The behavior of the angled cracked cylindrical shells under torsion is also studied. The general specifications of these shells are the same as non-cracked shells. A crack with a specified length is produced on the shell and the angle of the crack increases gradually. In each stage, the normalized value of reduction of cracked shell buckling capacity is calculated. When the crack angle is horizontal or oblique, the behavior of shell is like to that of the horizontally cracked shell. When the angle of crack is near  $90^\circ$ , the behavior of shell is like the vertically cracked shell.

The effect of Poisson's ratio on buckling load of a cracked cylindrical shell has been studied. In addition, the effect of cylinder length on buckling load has been studied. In some cases, different mesh generations for crack tips has been studied and checked. Curved panels with different curvature under shearing load have been studied too. The study of non-cracked curved

panels under shearing load in this research shows that if the curvature of the panel increases, the load carrying capacity of the panel increases.

## 7. CONCLUSIONS

It has been observed that in shells with circumferential cracks, the increase of crack length up to a certain limit, doesn't affect the buckling load and buckled shape of the cylinder. In other words, the behavior of the cracked shell remains the same as that of the non-cracked shell. When the crack length exceeds the critical limit, limit load begins to deteriorate.

In shells with axial crack, the increase of crack length up to a certain limit doesn't affect the buckled shape of the cylinder. When the crack length exceeds the critical limit, the buckling mode shape of the shell will become a local buckling mode near crack edges. This matter will cause a high decrease in the buckling load up to 1/2 of the non-cracked cylindrical shell buckling load. Thereafter, the buckling load will have little sensitivity to crack length and with increase in the crack length, there will only be a small decrease in the buckling load. It is also observed that the critical crack length in shells with circumferential and axial cracks is approximately the same.

In shells with angled crack, when the crack angle is near horizontal or oblique, the behavior of shell is like the horizontally cracked shell. When the angle of the crack approximates 90°, the behavior of shell is like the vertically cracked shell. The load carrying capacity factor of cylindrical shells increases as the thickness of the shell increases. On the other hand, as the thickness of cylindrical shell increases, the sensitivity of the shell to crack decreases.

The critical crack length for cylindrical shells under torsion is larger than that for cylindrical shells under axial compression. This means that axially compressed cylinders are more sensitive to the crack. The study of various methods for crack modeling in this research shows that the methods of crack edge modeling have little effect on the buckling load of shells.

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